

0.1 Two-dimensional Fourier transform imaging

A [two-dimensional Fourier transform](#) (2D-FT) is computed numerically or carried out in two stages, both involving ‘standard’, one-dimensional [Fourier transforms](#). However, the second stage Fourier transform is *not the inverse* Fourier transform (which would result in the original [function](#) that was transformed at the first stage), but a Fourier transform in a second variable– which is ‘shifted’ in value– relative to that involved in the result of the first Fourier transform. Such 2D-FT analysis is a very powerful method for three-dimensional reconstruction of polymer and biopolymer structures by [two-dimensional Nuclear Magnetic resonance \(2D-FT NMR](#), [1]) of solutions for molecular weights (M_w) of the dissolved polymers up to about 50,000 M_w . For larger biopolymers or polymers, more complex methods have been developed to obtain the desired resolution needed for the 3D-reconstruction of higher [molecular structures](#), e.g. for 900,000 M_w , methods that can also be utilized *in vivo*. The 2D-FT method is also widely utilized in optical spectroscopy, such as *2D-FT NIR Hyperspectral Imaging*, or in *MRI imaging* for research and clinical, diagnostic applications in Medicine.

A more precise mathematical definition of the ‘double’ Fourier transform involved is specified next, and a precise example follows the definition.

Definition 0.1. A 2D-FT, or two-dimensional Fourier transform, is a standard Fourier transformation of a function of two variables, $f(x_1, x_2)$, carried first in the first variable x_1 , followed by the Fourier transform in the second variable x_2 of the resulting function $F(s_1, x_2)$. (For further specific details and example for 2D-FT Imaging v. URLs provided in the following recent Bibliography).

Example 0.1 A 2D Fourier transformation and phase correction is applied to a set of 2D [NMR](#) (FID) signals $s(t_1, t_2)$ yielding a real 2D-FT [NMR ‘spectrum’](#) (collection of 1D [FT-NMR](#) spectra) represented by a [matrix](#) S whose elements are

$$S(\nu_1, \nu_2) = \mathbf{Re} \int \int \cos(\nu_1 t_1) \exp(-i\nu_2 t_2) s(t_1, t_2) dt_1 dt_2,$$

where ν_1 and ν_2 denote the discrete indirect double-quantum and single-quantum([detection](#)) axes, respectively, in the 2D NMR experiments. Next, the [covariance matrix](#) is calculated in the frequency [domain](#) according to the following equation:

$$C(\nu'_2, \nu_2) = S^T S = \sum_{\nu_1} [S(\nu_1, \nu'_2) S(\nu_1, \nu_2)],$$

with ν_2, ν'_2 taking all possible single-quantum frequency values and with the summation carried out over all discrete, double quantum frequencies ν_1 .

0.2 Example 0.2

[Atomic structure reconstruction by 2D-FT of STEM Images](#)(obtained at Cornell University) reveals the electron distributions in a high-temperature cuprate superconductor ‘paracrystal’; both the domains (or ‘location’) and the local symmetry of the “pseudo-gap” are seen in the electron-pair correlation band responsible for the high-temperature [superconductivity](#) effect .

0.3 Remarks

So far there have been three Nobel prizes awarded for 2D-FT NMR/MRI during 1992-2003, and an additional, earlier Nobel prize for 2D-FT of [X-ray](#) data (‘CAT scans’); recently the advanced possibilities of 2D-FT techniques in [Chemistry](#), Physiology and Medicine received very significant recognition.

References

- [1] Kurt Wütrich: 1986, *NMR of Proteins and Nucleic Acids.*, J. Wiley and Sons: New York, Chichester, Brisbane, Toronto, Singapore. ([Nobel Laureate in 2002 for 2D-FT NMR Studies of Structure and Function of Biological Macromolecules](#)); [2D-FT NMR Instrument Image Example: a JPG color image of a 2D-FT NMR Imaging ‘monster’ Instrument](#)
- [2] Richard R. Ernst. 1992. Nuclear Magnetic Resonance Fourier Transform (2D-FT) Spectroscopy. [Nobel Lecture](#), on December 9, 1992.
- [3] Peter Mansfield. 2003. [Nobel Laureate in Physiology and Medicine for \(2D and 3D\) MRI](#).
- [4] D. Benett. 2007. *PhD Thesis*. Worcester Polytechnic Institute. (*lots of 2D-FT images of mathematical, brain scans.*) [PDF of 2D-FT Imaging Applications to MRI in Medical Research](#).
- [5] Paul Lauterbur. 2003. [Nobel Laureate in Physiology and Medicine for \(2D and 3D\) MRI](#).
- [6] Jean Jeener. 1971. Two-dimensional Fourier Transform NMR, presented at an Ampere International Summer School, Basko Polje, *unpublished*. A verbatim quote follows from Richard R. Ernst’s Nobel Laureate Lecture delivered on December 2nd, 1992, “A new approach to measure two-dimensional (2D) spectra has been proposed by Jean Jeener at an Ampère Summer School in Basko Polje, Yugoslavia, 1971 ([6]). He suggested a 2D Fourier transform experiment consisting of two $\pi/2$ pulses with a variable time t_1 between the pulses and the time variable t_2 measuring the time elapsed after the second pulse as shown in Fig. 6 that expands the principles of Fig. 1. Measuring the response $s(t_1, t_2)$ of the two-pulse sequence and Fourier-transformation with respect to both time

variables produces a two-dimensional spectrum $S(O_1, O_2)$ of the desired form. This two-pulse experiment by Jean Jeener is the forefather of a whole class of $2D$ experiments that can also easily be expanded to multidimensional spectroscopy.”

- [7] A [2D-FT NMRI article](#) and Spectroscopy.
- [8] Cardiac infarct movies by [2D-FT NMR Imaging](#)